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How and when is dual trading irrelevant?

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Abstract

Within a general model of speculative trade, we derive the aggregate consequences of dual traders who process retail liquidity trades and trade on their own account. We prove that dual trading reduces total expected speculator profits unless speculators process all liquidity trade and trade with the same intensity on liquidity trade. In contrast, dual trading does not affect the information content of prices. We show how results generalize when we endogenize (a) speculator information via costly information acquisition about fundamentals or costly processing of liquidity trade, and (b) liquidity trader motives and welfare via endowment shocks.

JEL classification: G1; G14; D43

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1. Introduction

This paper introduces dual traders who both trade on their own accounts and process orders from retail customers to the standard dealership model of speculative trade in stock markets (Kyle, 1985). Dual trading is endemic to many asset markets (e.g., currency, commodity and futures markets) and there is a wide-spread sense that dual trading must harm liquidity traders, as speculators will exploit information about liquidity trade by taking partially offsetting positions, recognizing that liquidity trade drives prices away from true asset values.

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This paper shows that this intuition cannot be retrieved in static settings. We consider a general setting with arbitrary correlations in speculators' signals about asset values and arbitrary correlations in liquidity order flows. We prove that *if* speculators collectively observe all liquidity trade *and* speculators share the same trading intensity on liquidity trade, then speculative profits are the same if they see liquidity trade as if they do not.[1] The condition that speculators share the same trading intensity on liquidity order flow requires that each speculator's forecast of unfilled liquidity trade (liquidity trade net of speculative order flow based on liquidity trade) be the same. This is an incredibly demanding condition, one that also implies that speculative trade on liquidity order flow is *perfectly* correlated with total liquidity order flow.

We then prove that this irrelevance result *requires* this restrictive structure: a simple application of the Cauchy–Schwarz inequality reveals that unless total speculative trade on liquidity order flow is perfectly correlated with total liquidity order flow, total expected speculator profits are *always reduced* by seeing liquidity trade. Most dramatically, if speculators process some, but not all, liquidity trade, then liquidity trade not handled by speculators is effectively liquidity trade handled with a trading intensity of zero—effective trading intensities on liquidity trade then differ, breaking the required perfect correlation and resulting in lower expected speculative profits.

This result *only* depends on the functional form of speculator linear trading strategies, in particular that the speculator trades on liquidity order flow are unaffected by the market maker's pricing parameter λ, while their trades on asset fundamentals are scaled by $1/\lambda$. In particular, we show that as long as pricing is consistent with speculator trading strategies (i.e., that the market maker correctly extracts information about asset fundamentals via his filtering of net order flow), total speculator profits are reduced by a failure by speculators to coordinate on a common trading intensity on liquidity order flow. Indeed, we illustrate that total speculator profits fall monotonically as their trading intensities on liquidity trade diverge. In turn, since expected aggregate liquidity trader losses equal expected speculator profits via the market maker's zero expected profit condition, it follows that dual trading reduces expected total liquidity trader losses.

One might then conjecture that since speculators are hurt by the opportunity to dual trade via the equilibrium impact on pricing, it must be that more information ends up being revealed through prices. Remarkably, we find that this is not so: even though dual trading generically lowers expected speculator profits, it has *no* effect on the residual forecast error uncertainty in prices. In particular, the correlation structure of liquidity trade does not affect the efficiency properties of prices—there is no "informational cost" to dual trading.

We next investigate how outcomes are affected when we endogenize speculator information about asset fundamentals and the amount of liquidity trade that the speculator processes. We first show that dual trading induces speculators to acquire less information about fundamentals—dual trading increases the price impact of order flow, reducing the value of private information—implying that the negative consequences of dual trading for speculator profits are exacerbated when information is endogenized via costly acquisition. In particular, even if the conditions for the irrelevance result hold when speculators are exogenously endowed with information, total expected speculator profits are reduced by seeing liquidity trade if we endogenize information acquisition.

So, too, we establish a negative result when we consider a monopolist speculator who must decide how much costly independently-distributed liquidity order flow to process.

[1]This result extends Rochet and Vila's (1994) finding for a single informed speculator.

When the speculator is exogenously endowed with a share of liquidity order flow, his profits are a U-shaped function of this share, maximized when he handles none or all liquidity order flow. In sharp contrast, when we endogenize the portion of liquidity order flow that the speculator processes, we find that his expected profits decline monotonically with the fraction that he processes. In particular, *reductions* in the cost of processing liquidity order flow *hurt* the speculator provided that those costs are not so low that the speculator handles all liquidity trade.

Our final step is to endogenize liquidity trade à la Spiegel and Subrahmanyam (1992) by considering risk-averse liquidity traders with mean–variance preferences over final wealth who receive endowment shocks: one type of liquidity trader submits orders indirectly via a monopolist dual-trading speculator, while the other submits orders directly to the market maker. This analysis is important, because one suspects that the welfare impact of dual trading is unevenly divided between liquidity traders, as the liquidity trader who trades via the speculator gains from having the price impact of his order partially offset, while the other liquidity trader's trading costs rise because prices become more sensitive to order flow.

We show the ways in which this suspicion is misplaced. We first derive analytically a critical level of risk aversion that determines which liquidity agent offsets a greater fraction of his endowment shock: the liquidity trader who submits his order via the speculator faces a reduced price impact because half of his order is offset by the speculator (ceteris paribus, increasing his order), but he also faces more uncertainty about the execution price of his order due to his greater exposure to "liquidity trade" risk (ceteris paribus, reducing his order). For risk aversion levels below the critical cut-off, the liquidity trader who trades with the dual trader has a higher trading intensity; while for higher levels of risk aversion, the agent trading directly with the market maker has a higher trading intensity.

Quite strikingly, we prove that at this critical risk aversion level, the welfare of a liquidity trader does *not* depend on whether he trades indirectly via the dual trader, or directly via the market maker. Moreover, we prove that at this critical level of risk aversion, liquidity trader welfare is always higher when dual trading is feasible than when it is banned. In sharp contrast, the opportunity to dual trade hurts speculators: we prove that the speculator profits are reduced even when he processes orders from a risk-tolerant liquidity trader who submits orders more aggressively due to dual trading.

This welfare invariance result does *not* extend to other risk aversion levels. Obviously, in the neighborhood of the critical risk aversion level, both liquidity trader types continue to gain from dual trading. More generally, we establish numerically that controlling for the endowment shock variance, the difference in liquidity trader welfare is proportional to the coefficient-of-risk-aversion-weighted difference in their endogenous trading intensities: for lower levels of risk aversion, the liquidity trader who trades with the dual trader trades more aggressively and derives a higher ex-ante expected utility, while for risk aversions above the critical level, the agent trading directly with the market maker does better. Indeed, while some liquidity traders always benefit from the presence of the dual trader, for risk aversions far from the critical level, one type can be hurt.

1.1. The dual trading literature

Roell (1990) is the first to consider dual trading. She shows that with a monopolist speculator who knows the asset's value and broker-dealers who observe equal shares of independently-distributed liquidity trade, liquidity trader losses are reduced by the observability of liquidity trade if the broker-dealers do not handle all liquidity trade. Our analysis proves that this result is quite general.

Sarkar (1995) considers variants of Roell's (1990) independently-distributed liquidity trade model. Sarkar considers a single broker who processes all order flow, *both* speculative and liquidity trade, distinguishing between them, and who can trade on this information. Speculators internalize the broker's presence, and when selecting order flow weigh the information released (trade less) versus their first-mover stature (trade more). Sarkar finds that dual trading has no impact on expected speculator profits. Our analysis reveals why he obtains this irrelevance result: interpreting the broker as another speculator, in essence, Sarkar's setting becomes the non-generic one where there is a single trading intensity on liquidity order flow. Sarkar continues on to endogenize liquidity trade order flow and derives an analogous welfare irrelevance result—dual trading has no impact on liquidity trader welfare. Our welfare analysis shows that this irrelevance result also hinges on the non-generic setting.

Finally, Fishman and Longstaff (1992) develop a very different dual-trading model in a Glosten and Milgrom (1985) style setting, where if traders trade, they submit round lot orders. The broker receives orders from up to two traders, who have random probabilities of being informed about the asset's value (the realization of the probability is observed by the broker, but not whether the trader is, in fact, informed). The broker then updates about the likelihood a trade is information based, and when two orders do not arrive can follow-up his customer's order with one on his own behalf to a competitive market maker.[2] Fishman and Longstaff close the model by assuming that it is costly to provide brokerage services, and that the fee for processing orders earns the broker zero profits: with dual trading, commissions are reduced, but the price impact is widened. Since everyone is risk neutral, in the aggregate, customers are unaffected by dual trading; but the composition of who gains is affected—customers who tend to be uninformed gain from dual trading at the expense of informed customers, as they benefit from the reduced commission and are not hurt by the follow-up trading. Empirically, Fishman and Longstaff find that in futures markets, dual traders earn higher profits than non-dual traders, and their uninformed customers do better as well in terms of execution costs. Both of these results are consistent with our model. Fishman and Longstaff do not provide a broader welfare analysis; our analysis indicates that the welfare impacts of dual trading on both types of liquidity traders hinge on their level of risk aversion, and we provide conditions under which all uninformed liquidity traders benefit.

2. The base model

There is a single risky asset with liquidation value $v = v_0 + \sum_{i=1}^{N+1} e_i$, where v_0 is public information, and $(e_1,\dots,e_{N+1})$ are drawn from a multivariate normal distribution with zero mean and arbitrary variance–covariance matrix Ψ_0. There are N risk-neutral speculators. Speculator i sees e_i, $i=1,\dots,N$, and e_{N+1} (possibly degenerate) is observed by no agent. Thus, the framework accommodates arbitrary numbers of speculators with arbitrary signal correlations.

In addition to speculative trade, there is uninformed liquidity trade. Total liquidity trade is given by $\sum_{i=1}^{N+1} u_i$, where $(u_1, \dots, u_{N+1})$ are drawn from a multivariate normal

[2] Three orders would reveal the broker is trading on information.

distribution with zero mean and variance–covariance matrix Λ_0. The N+1 components of liquidity trade allow each speculator to have private information (possibly degenerate) about liquidity trade, and there can be additional liquidity trade that no speculator observes. We assume only that liquidity trade is distributed independently from the innovations to the asset's value.

The agents trade in a market made by risk-neutral competitive market makers who only see the net order flow, and set price equal to the expected value of the asset given the net order flow. Finally, after trading occurs, the asset's liquidation value v becomes public information.

We conjecture and verify that speculators adopt trading strategies that are linear functions of their private information, and that price is a linear function of the net order flow. Thus, if ω is the net order flow from informed and liquidity traders, pricing takes the form

$$p(\omega) = v_0 + \lambda\omega.$$

Lemma 1. *Trading strategies of speculators take the form*

$$x_i(e_i, u_i) = \frac{s_i}{\lambda} e_i - a_i u_i.$$

Further, $a_i = a_j = a$, *for all* $i,j = 1,\ldots, N$ *if and only if the signal-to-noise ratio in liquidity trade is the same for all speculators, i.e.,* $\sum_k cov(u_i, u_k)/var(u_i) = \sum_k cov(u_j, u_k)/var(u_j)$.

Proof. A speculator i who observes $\{e_i, u_i\}$ chooses x_i to maximize expected trading profits:

$$\max_{x_i} E\left[x_i\left(\sum_j e_j - \lambda\left(\sum_j x_j + \sum_j u_j)\right)|e_i, u_i\right].$$

The associated first-order condition characterizing the optimal order is

$$0 = E\left[\sum_j e_j - \lambda\left(\sum_j x_j + \sum_j u_j\right) - \lambda x_i | e_i, u_i\right].$$

Re-arranging and substituting for the conjectured linear forms of strategies yields

$$x_i = \frac{\left[E\left[\sum_j e_j|e_i\right] - E\left[\sum_{j\neq i} s_j e_j|e_i\right]\right]}{2\lambda} - \frac{\left[E\left[\sum_j u_j|u_i\right] - E\left[\sum_{j\neq i} a_j u_j|u_i\right]\right]}{2}. \tag{1}$$

Since $E[e_j|e_i] = (cov(e_i, e_j)/var(e_i))e_i$ and $E[u_j|u_i] = (cov(u_j, u_i)/var(u_i))u_i$, trading strategies are linear. Hence, order flow is normally distributed and pricing is linear. Solving for a_i yields

$$a_i = \sum_{k=1}^{N} \frac{cov(u_i, u_k)}{var(u_i)}(1 - a_k). \tag{2}$$

Condition (1) says that speculator i's trade on liquidity trade of $a_i u_i$ corresponds to his best forecast of the remaining unfilled liquidity order flow, *after* accounting for the impact of his order. To see that $a_i = a_j = a$ demands identical signal-to-noise ratios, substitute and solve for

$$\frac{a}{1-a} = \sum_k \frac{cov(u_i, u_k)}{var(u_i)}. \qquad \square$$

Our proposition characterizes how speculators' ability to observe liquidity trade affects aggregate outcomes, interpreting "unobserved liquidity trade" as that handled by speculator $N+1$ who chooses $a_{N+1}=0$.

Proposition 1. *Suppose that all speculators $i,j \in \{1,\ldots,N+1\}$ with $var(u_i), var(u_j)>0$ share the same trading intensity on liquidity trade, i.e., $a_i=a_j=a$. Then total expected speculator profits equal those when speculators do not see liquidity trade.*

Suppose, instead, that some speculators $i,j \in \{1,\ldots,N+1\}$ with $var(u_i), var(u_j)>0$ differ in their trading intensities on liquidity trade, i.e., $a_i \neq a_j$. Then total expected speculator profits are strictly lower than they would be if speculators did not see liquidity trade.

Proof. If speculator i sees only e_i, the market maker's zero expected profit condition is

$$\begin{aligned} 0 &= E\left[\left(\sum_i \left(\frac{s_i}{\lambda}e_i + u_i\right)\right)\left(\sum_i \left(e_i - \lambda\left(\frac{s_i}{\lambda}e_i + u_i\right)\right)\right)\right] \\ &= \frac{1}{\lambda}E\left[\left(\sum_i s_i e_i\right)\left(\sum_j (e_j - s_j e_j)\right)\right] - \lambda E\left[\left(\sum_i u_i\right)\left(\sum_j u_j\right)\right]. \end{aligned} \tag{3}$$

Define

$$A \equiv \sum_j s_j e_j \quad \text{and} \quad B \equiv \sum_j (e_j - s_j e_j)$$

and

$$X \equiv \sum_j u_j \quad \text{and} \quad Y \equiv \sum_j (1-a_j)u_j.$$

Then we can write covariances as inner products:

$$E[AB] = \langle A, B \rangle \quad \text{and} \quad E[X^2] = \langle X, X \rangle$$

and so on. Then, we can write the market maker's zero expected profit condition (3) compactly as

$$\frac{1}{\lambda}\langle A, B \rangle = \lambda \langle X, X \rangle. \tag{4}$$

The left-hand side of (4) is expected total speculator profits. The right-hand side is expected total liquidity trader losses. Solving (4) for the equilibrium λ yields: $\lambda = \langle A, B \rangle^{1/2} / \langle X, X \rangle^{1/2}$. Substituting for λ into either side of (4) yields total expected equilibrium speculator profits,

$$E\left[\sum_i \pi_i\right] = \langle A, B \rangle^{1/2} \langle X, X \rangle^{1/2}.$$

If, instead, speculator i sees both e_i and u_i, the market maker's zero profit condition becomes

$$0=E\left[\left(\sum_i\left(\frac{s_i}{\lambda}e_i+(1-a_i)u_i\right)\right)\left(\sum_i e_i-\lambda\left(\frac{s_i}{\lambda}e_i+(1-a_i)u_i\right)\right)\right].$$

The zero-profit condition for the market maker is then

$$\frac{1}{\lambda}\langle A,B\rangle=\lambda E\left[\left(\sum_i(1-a_i)u_i\right)\left(\sum_j(1-a_j)u_j\right)\right]=\lambda\langle Y,Y\rangle.$$

We solve the zero-profit condition for $\lambda=\langle A,B\rangle^{1/2}/\langle Y,Y\rangle^{1/2}$ and substitute it into expected liquidity trader losses to obtain

$$\begin{aligned}E\left[\sum_j\pi_j\right]&=E\left[\lambda\left(\sum_j(1-a_j)u_j\right)\left(\sum_i u_i\right)\right]=\lambda\langle Y,X\rangle\\&=\frac{\langle A,B\rangle^{1/2}}{\langle Y,Y\rangle^{1/2}}\langle Y,X\rangle\leq\frac{\langle A,B\rangle^{1/2}}{\langle Y,Y\rangle^{1/2}}\langle Y,Y\rangle^{1/2}\langle X,X\rangle^{1/2}\\&=\langle A,B\rangle^{1/2}\langle X,X\rangle^{1/2}.\end{aligned}$$

The inequality follows from the Cauchy–Schwarz inequality and is strict unless symmetry holds, $a_i=a_j=a$, $\forall i,j$ who handle liquidity trade, i.e., for whom it is not the case that $u_k\equiv 0$. Such k do not enter the inner product as $E[(1-a_k)\,(1-a_j)\,u_k\,u_j]=E\,[(1-a_k)\,(1-a_j)\,u_j]=0$. When trading intensities are the same for speculators who handle trade, the $1-a$ factors out, so that $\langle Y,Y\rangle=(1-a)^2\langle X,X\rangle$, leaving expected profits unchanged. □

To see the result, divide through the inequality above by $\langle Y,Y\rangle^{1/2}\langle X,X\rangle^{1/2}$, and write it as

$$\frac{\langle A,B\rangle^{1/2}}{\langle Y,Y\rangle^{1/2}}\frac{\langle Y,X\rangle}{\langle Y,Y\rangle^{1/2}\langle X,X\rangle^{1/2}}\leq\frac{\langle A,B\rangle^{1/2}}{\langle Y,Y\rangle^{1/2}}.$$

This inequality is strict *unless* the correlation coefficient $\langle Y,X\rangle/\langle Y,Y\rangle^{1/2}\langle X,X\rangle^{1/2}$ is one, i.e., unless total speculative trade on liquidity order flow is perfectly correlated with total liquidity order flow, which requires that there be no variation in speculative trading intensities on liquidity order flow. That is, to obtain a profit irrelevance result, speculators must share the same trading intensity on liquidity trade (i.e., they must process all liquidity trade, and the signal-to-noise ratio in liquidity order flow must be the same for each).

It is worth emphasizing the generality of this result. We impose no restrictions on the correlations or magnitudes of private information that speculators have about fundamentals, assuming only that liquidity trade is distributed independently of asset fundamentals.[3] The result is starkest when we alter the environment of Rochet and Vila

[3]Note that taking λ as exogenous, trading on private information about liquidity trade has no impact on speculative trading on fundamentals: fixing λ, profits are additively separable between profits from fundamentals and profits from information about liquidity trade. As Proposition 1 highlights, this means that aggregate outcomes do not depend on which speculators have private information about asset fundamentals and liquidity trade, which ones only have information about fundamentals, and which ones only have information about liquidity trade. While individual speculator profits hinge on who has what information, from an aggregate perspective, total speculator profits would be the same if we "split" the speculators with information about both

(1994), and consider a single speculator who processes some liquidity trade, but not all: that single speculator would be strictly better off if he could commit to not processing *any* liquidity trade.

This result only characterizes the consequences for total expected speculator profits, and via the zero profit constraint for the market maker, the total expected losses of liquidity traders. Dual trading alters the source of speculator profits, as speculators now earn profits from handling liquidity trade, but reduced profits from trades on intrinsic information about asset fundamentals. Moreover, the gains and costs are not evenly divided—speculators who handle liquidity trade gain that profit, while all share the common damage to profits due to intrinsic information due to the increased sensitivity of price to order flow.[4] So, too, liquidity traders who trade via the dual trader gain from having some of the price impact of their order flows offset, but liquidity traders whose orders go directly to the market maker face higher expected execution costs, as the price impact of their order is greater due to the increase in λ, and dual traders do not mitigate this impact by offsetting some of their order. This result highlights the importance of endogenizing liquidity trade, to provide a more thorough account of how the welfare effects of dual trading extend more widely to all liquidity traders.

To provide the full intuition for Proposition 1 most clearly, it first helps to derive the impact of dual trading on the information content of prices, i.e., the variance of the forecast error in pricing. Proposition 2 reveals the surprising result that even though speculator profits vary sensitively with the correlation structure of liquidity trade when there is dual trading, the information content of pricing does *not* vary with the correlation structure of liquidity trade, and, in particular, with whether there is dual trading. That is, even though dual trading generically lowers expected speculator profits, it has *no* effect on the residual forecast error uncertainty in prices.

Proposition 2. *The information content of prices is unaffected by the presence of dual trading.*

Proof. The forecast error variance is

$$E\left[\left(\sum_i e_i-\lambda\left(\frac{\sum_i s_i e_i}{\lambda}+\sum_j u_j(1-a_j)\right)\right)^2\right]=E\left[\left(\sum_i((1-s_i)e_i)-\lambda\left(\sum_j u_j(1-a_j)\right)\right)^2\right]$$

$$=E[(B-\lambda Y)^2]=\langle B,B\rangle-2\lambda\langle B,Y\rangle+\lambda^2\langle Y,Y\rangle=\langle B,B\rangle+\langle A,B\rangle,$$

where the final equality follows from $cov(B,Y)=0$, and $\lambda=[\langle A,B\rangle/\langle Y,Y\rangle]^{0.5}$. □

Thus, Proposition 2 reveals that dual trading has no effect on the efficiency properties of prices: the forecast error variance in price is unaffected by the informational structure of

(*footnote continued*)
fundamentals and liquidity trade into two, so that one had private information about fundamentals, and the other had information about liquidity trade.

[4]In particular, if the liquidity trades that speculators observe are independently (but not identically) distributed, then $a_i=\frac{1}{2}$ for all i, and speculators who handle greater shares of liquidity trade gain at the expense of other speculators: each individual speculator profits from handling a greater share of liquidity trade, but such gains are exactly offset by the reduction in the share and hence profits of other speculators. Thus, our analysis is consistent with the empirical finding by Fishman and Longstaff (1992) that in futures markets, dual traders earn higher profits than non-dual traders.

liquidity trade, and by whether or not there is dual trading. Indeed, the forecast error variance $\langle B,B\rangle + \langle A,B\rangle$ simplifies to $\langle \sum_j e_j, \sum_j e_j\rangle - \langle \sum_j e_j, \sum_j s_j e_j\rangle$, i.e., it always equals the variance of the fundamentals less the information revealed via direct speculation on fundamentals (recall that s_j is scaled by λ in speculator trading intensities). This result indicates that not only does dual trading reduce the expected losses of liquidity traders, but that there is no offsetting "informational cost," as prices are just as informative.

It is important to understand what drives the results in Propositions 1 and 2. These results hinge only on the *form* of the speculators' linear trading strategies, in particular that (a) λ does not affect trade on liquidity order flow by speculators, and (b) that speculator trade on private information about asset fundamentals is scaled by $1/\lambda$. In all other respects, Propositions 1 and 2 hold independently of optimization by speculators, and only require consistent information extraction from net order flow by the market maker, i.e., that λ reflects/incorporates the reduction in liquidity order flow due to trade by speculators. In equilibrium, of course, given λ, speculators' trade on liquidity order flow is optimal, but the content of Propositions 1 and 2 do not hinge on this. Proposition 2 then follows because given the consistent λ, the scaling by speculators of their trades on asset fundamentals by $1/\lambda$ ensures that the information content of net order flow is unaffected by how speculators trade on liquidity order flow. As a result, the forecast error variance in pricing is unaffected by speculator trade on liquidity order flow.

What underlies Proposition 1 is the fact that given pricing is consistent with trading strategies (i.e., that the market maker unwinds the optimization by speculators via his filtering of net order flow), total speculator profits are reduced by a failure by speculators to coordinate on a common trading intensity on liquidity order flow. To see how and why it is only the coordination of speculator trading intensities that is relevant, we again pose speculator profits in terms of the expected losses of liquidity traders to the market maker,

$$E\left[\lambda\sum_j(1-a_j)u_j\sum_i u_i\right] = \frac{[\langle A,B\rangle]^{1/2}}{\left(E\left[\sum_j(1-a_j)u_j\sum_i(1-a_j)u_i\right]\right)^{1/2}}\left[\sum_j(1-a_j)u_j\sum_i u_i\right].$$

If the a_j's are the same, then profit is

$$[\langle A,B\rangle]^{1/2}\frac{(1-a)E\left[\sum_j u_j\sum_i u_i\right]}{\left((1-a)^2E\left[\sum_j u_j\sum_i u_i\right]\right)^{1/2}}$$

and the $(1-a)$ terms cancel. That is, if speculators remove any constant fraction a of the liquidity trade variance from the order flow, then market makers only receive fraction $1-a$ of order flow and respond by scaling up λ by $1/(1-a)$. Because speculators scale trading intensities on asset fundamentals by $1/\lambda$, this higher λ causes speculators to reduce trading intensities on intrinsic information by $1-a$ percent, leaving the ratio of trade on intrinsic information to net liquidity trade unaffected (i.e., the signal-to-noise ratio in net order flow is unaffected). The consequence is that expected liquidity trade losses are unaffected, as the reduction in liquidity trade variance due to speculator trade on it is exactly offset by the increase in λ.[5]

[5] Rochet and Vila (1994) prove an analogous irrelevance result for a single speculator who sees all order flow. Their finding does not require that private information and liquidity trade be normally distributed. With the

But what happens when speculators fail to coordinate their trading intensities on liquidity trade? To illustrate the negative consequences most clearly, consider a setting in which speculator 1 receives fraction α of the independently-distributed liquidity trade variance and offsets fraction a_1 of liquidity trade, and speculator 2 receives fraction $1-\alpha$, and offsets fraction a_2.[6] Then expected liquidity trader losses when pricing is consistent with a_1 and a_2, i.e., when the market maker correctly extracts information from the net order flow, are

$$\lambda[\alpha(1-a_1)+(1-\alpha)(1-a_2)]\sigma_u^2 \propto \frac{[\alpha(1-a_1)+(1-\alpha)(1-a_2)]\sigma_u^2}{[\alpha(1-a_1)^2\sigma_u^2+(1-\alpha)(1-a_2)^2\sigma_u^2]^{1/2}}, \tag{5}$$

where the denominator is the liquidity trade term in the signal-to-noise ratio composition of λ. Note that the numerator is linear in α, while α enters the denominator convexly via λ, which immediately indicates that profits *must* vary with α. To see that profits, in fact, fall when speculators do not coordinate on a common trading intensity, without loss of generality, write a_1 and a_2 as a mean preserving spread around a common trading intensity, a, i.e.,

$$a_1 = a-\frac{\Delta}{\alpha} \quad \text{and} \quad a_2 = a+\frac{\Delta}{1-\alpha}.$$

Then, the right-hand side of Eq. (5) becomes

$$\frac{\left[\alpha\left(1-a+\dfrac{\Delta}{\alpha}\right)+(1-\alpha)\left(1-a-\dfrac{\Delta}{1-\alpha}\right)\right]\sigma_u^2}{\left[\alpha\left(1-a+\dfrac{\Delta}{\alpha}\right)^2\sigma_u^2+(1-\alpha)\left(1-a-\dfrac{\Delta}{1-\alpha}\right)^2\sigma_u^2\right]^{1/2}}$$
$$=\frac{[(1-a)]\sigma_u}{\left[\alpha\left(1-a+\dfrac{\Delta}{\alpha}\right)^2+(1-\alpha)\left(1-a-\dfrac{\Delta}{1-\alpha}\right)^2\right]^{1/2}}.$$

The numerator—the fraction of the standard deviation of liquidity trade that is handled by the market maker—is obviously unaffected by a mean-preserving spread in how speculators trade on it, but the denominator is an increasing function of the spread term, Δ. To see this, observe that the derivative of the denominator term

$$\alpha\left(1-a+\frac{\Delta}{\alpha}\right)^2+(1-\alpha)\left(1-a-\frac{\Delta}{1-\alpha}\right)^2,$$

with respect to $\Delta>0$ is

$$2\left[\frac{\Delta}{\alpha}+\frac{\Delta}{1-\alpha}\right]>0.$$

Hence, expected speculator profits decline monotonically as their trading intensities diverge.

(*footnote continued*)
added assumption of normality, the proposition generalizes Rochet and Vila to multi-speculator contexts, where speculators process all order flow and share the same trading intensity on liquidity trade.

[6] In equilibrium with a single speculator 1 and liquidity trade via speculator 2 goes directly to the market maker we have $a_1=\frac{1}{2}$, and $a_2=0$. However, as we have discussed, the particular values are irrelevant.

2.1. Can endogenous information acquisition overturn our result?

We now show that even in the setting of Proposition 1, where speculators share the same trading intensity on liquidity trade, the limited irrelevance result obtained there vanishes when we endogenize information acquisition by speculators. To do this, we suppose that speculators, rather than being *exogenously* endowed with information about the asset's value, must *endogenously* acquire this information at a cost. We then show that with dual trading, speculators endogenously choose to acquire strictly less information than they would if they did not process liquidity trade. It immediately follows that dual trading strictly reduces expected speculator profits.

To show this most starkly, we consider a monopolist speculator. Then, absent the endogeneity of information acquisition, Rochet and Vila (1994) provide a very general, distribution-free, irrelevance result—for expected aggregate outcomes, it does not matter whether the speculator observes liquidity trade. In particular, a monopolist speculator who sees only a noisy signal of the asset's value expects the same profits as he would were he also to see all liquidity order flow.

With endogenous information acquisition, the monopolist speculator must choose the costly quality of his signal to acquire about the asset. Specifically, at a cost of $c(\tau)$, the speculator can obtain a noisy signal $s = v + \varepsilon$ of the asset's value v, where $v \sim N(0, \sigma^2)$, and ε is independently and normally distributed with mean zero and precision τ. The signal's cost $c(\tau)$ is a strictly convex, increasing function of its precision, τ, with $c(0) = 0, c'(0) = 0$ and $\lim_{\tau \to \infty} c(\tau) = \infty$.

We have established that the profits that the speculator expects from trading on intrinsic information enter separably from the profits from trading on liquidity order flow. It follows that the speculator's optimal choice of signal quality solves

$$\max_{\tau}[E[\pi_v(\tau, \lambda)] + E[\pi_L(\lambda)] - c(\tau)],$$

where $E[\pi_v(\tau, \lambda)]$ is his expected profit from trading on intrinsic information about the asset, and $E[\pi_L(\lambda)]$ is his expected profit from trading on information about liquidity orders if he is a dual trader. Observe that the signal quality that the speculator acquires affects profits from dual trading only indirectly via the equilibrium pricing parameter, λ, which the speculator takes as exogenous. Phrased differently, the opportunity to dual trade on knowledge of liquidity order flow affects the speculator's choice of signal quality *only* via λ. Therefore, the speculator's optimization problem simplifies to

$$\max_{\tau}[E[\pi_v(\tau, \lambda)] - c(\tau)].$$

Substituting for $E[\pi_v(\tau, \lambda)] = (\sigma^4 \tau)/4\lambda(\sigma^2 \tau + 1)$,[7] we rewrite the speculator's optimization problem as

$$\max_{\tau} \left[\frac{\sigma^4 \tau}{\sigma^2 \tau + 1} - 4\lambda c(\tau) \right].$$

[7]We remind the reader that the choice of trading intensity on the signal is made ex post, conditional on the signal, entailing a linear projection $(\sigma^2/(\sigma_e^2 + 1/\tau))(v + \varepsilon)$, but the choice of signal quality is taken ex-ante with an unconditional expectation, taking the projection as given, and this yields the formula for profit.

The first-order condition characterizing the optimal signal quality is

$$4\lambda c'(\tau) = \frac{\sigma^4}{(\sigma^2\tau + 1)^2}.$$

Inspection reveals that the greater is λ, the noisier is the signal that the speculator acquires. We have shown that dual trading causes market makers to scale up λ to account for the reduced liquidity trade that they receive. As a result, a dual-trading speculator acquires a strictly noisier signal.

In equilibrium, λ will reflect the reduced signal quality that dual traders acquire, but λ cannot fall to the point where asset signal qualities are the same with and without dual trading. To see this, observe that for the signal qualities to be the same, λ must be the same; but a market maker who expects zero profits from λ absent dual trading, must then expect negative profits with dual trading as he only receives a fraction of the liquidity trade, a contradiction.

This analysis establishes that a dual-trading monopolist speculator acquires a strictly noisier signal about the asset's value.[8] Since expected profits are increasing in the quality of the asset's signal, we can exploit the profit equivalence result in Proposition 1 for identical signal qualities to conclude that the opportunity to dual trade reduces the monopolist speculator's expected equilibrium profits, and hence reduces expected liquidity trader losses. Thus, even when with exogenous information, liquidity traders are indifferent to dual trading, with endogenous information acquisition, liquidity traders prefer a dual trading environment.

2.2. *What about costly acquisition of liquidity order flow?*

We now contemplate how outcomes are affected when we endogenize the amount of liquidity trade that a speculator handles. In order to make explicit calculations of profits, we consider a monopolist speculator who at a cost of $c(m) = cm^2$ can process fraction m of the independently-distributed liquidity trade: the speculator handles liquidity order flow u_1, which is distributed $N(0, m\sigma_u^2)$, while the remaining independently-distributed liquidity order flow, $u_2 \sim N(0, (1-m)\sigma_u^2)$, goes directly to the market maker. Thus, the total variance of the liquidity trade is

$$\langle X, X \rangle = m\sigma_u^2 + (1-m)\sigma_u^2 = \sigma_u^2.$$

We recall that the separability between profits from handling liquidity trade and from observing asset fundamentals means that we can treat information acquisition about fundamentals as exogenous. Accordingly, we assume that the monopolist speculator observes the asset value $e_1 \sim N(0, \sigma_e^2)$ perfectly.

We have established in this setting that the speculator adopts trading strategy,

$$x_1 = \frac{e_1}{2\lambda} - \frac{u_1}{2},$$

[8]It follows that endogenizing information acquisition by the speculator undoes the price efficiency irrelevance result, Proposition 2, which established that for a *given* signal quality, prices contained the same amount of information independently of whether there was dual trading. With endogenous information acquisition about fundamentals, dual trading leads to strictly less information acquisition about asset fundamentals, so that prices are less informative.

and that the equilibrium pricing function is

$$\lambda^* = \left[\frac{\frac{\sigma_e^2}{4}}{\frac{m\sigma_u^2}{4} + (1-m)\sigma_u^2}\right]^{0.5} = \left[\frac{\sigma_e^2}{(4-3m)\sigma_u^2}\right]^{0.5}.$$

Were the speculator *exogenously* and *costlessly* endowed with a fraction m of liquidity order flow, then his expected profits are a U-shaped function of m. In particular, Proposition 1 implies that profits are maximized by $m=0$ and $=1$:

$$E[\pi_e(\lambda)] + E[\pi_u(m,\lambda)] = \frac{\sigma_e^2}{4\lambda} + \frac{m\sigma_u^2\lambda}{4} = \frac{\sigma_e\sigma_u}{4}\left[(4-3m)^{0.5} + \frac{m}{(4-3m)^{0.5}}\right]$$
$$= \frac{\sigma_e\sigma_u}{4}\frac{4-2m}{(4-3m)^{0.5}}.$$

Differentiation reveals that when the speculator exogenously and costlessly processes fraction m of liquidity order flow variance that his expected profits are minimized by $m = \frac{2}{3}$:

$$0 = \frac{d\frac{4-2m}{(4-3m)^{0.5}}}{dm} = \frac{-2(4-3m)^{0.5} + \frac{3}{2}\frac{(4-2m)}{(4-3m)^{0.5}}}{4-3m} \Rightarrow 8-6m = 6-3m \Rightarrow m = \frac{2}{3}.$$

However, when liquidity trade is costly to process, the speculator takes the price impact λ as exogenous when choosing how much liquidity trade to process, solving

$$\max_{m\in[0,1]}[E[\pi_e(\lambda)] + E[\pi_u(m,\lambda)] - c(m)] = \max_m \left[\frac{\sigma_e^2}{4\lambda} + \frac{m\sigma_u^2\lambda}{4} - cm^2\right].$$

Assuming an interior solution (i.e., $m<1$), the associated first-order condition is

$$2cm = \frac{\sigma_u^2\lambda}{4} = \frac{\sigma_e\sigma_u}{4(4-3m)^{0.5}} \Rightarrow cm = \frac{\sigma_e\sigma_u}{8(4-3m)^{0.5}} \Rightarrow \frac{\sigma_e^2\sigma_u^2}{64c^2} = m^2(4-3m).$$

The relevant root of the cubic is the largest one. The speculator handles all liquidity order flow if at $m=1$ the marginal benefit, $\sigma_u^2\lambda/4$, exceeds the marginal cost, $2cm$, which at $m=1$ occurs if $c<\sigma_u\sigma_e/8$.

The central question of interest is: how do the speculator's expected profits vary with the costs c of processing order flow? To address this question, we use $cm = \sigma_e\sigma_u/8(4-3m)^{0.5} \Rightarrow cm^2 = \sigma_e\sigma_u m/8(4-3m)^{0.5}$ to write the speculator's expected profits as

$$E[\pi_e(\lambda)] + E[\pi_u(m,\lambda)] - cm^2 = \frac{\sigma_e^2}{4\lambda} + \frac{m\sigma_u^2\lambda}{4} - \frac{\sigma_e\sigma_u m}{8(4-3m)^{0.5}}$$
$$= \frac{\sigma_e\sigma_u}{4}\frac{4-2m}{(4-3m)^{0.5}} - \frac{\sigma_e\sigma_u m}{8(4-3m)^{0.5}}$$
$$= \frac{\sigma_e\sigma_u}{8}\frac{8-5m}{(4-3m)^{0.5}}.$$

Now, differentiation reveals that when the speculator must endogenously acquire liquidity order flow at a cost, expected profits are monotonically declining in m:

$$\frac{d\frac{8-5m}{(4-3m)^{0.5}}}{dm}=\frac{-5(4-3m)^{0.5}+\frac{3}{2}\frac{(8-5m)}{(4-3m)^{0.5}}}{4-3m}=\frac{-5(4-3m)+\frac{3}{2}(8-5m)}{(4-3m)^{1.5}}=\frac{-8+\frac{15}{2}m}{(4-3m)^{1.5}}<0.$$

As the speculator processes more liquidity order flow when it is cheaper to acquire, it follows that expected speculator profits are monotonically *increasing* in c for $c>\sigma_e\sigma_u 243/16\,384$. That is, as long as costs are not so low that the speculator processes all liquidity order flow, reductions in the cost of liquidity order flow processing necessarily *hurt* the speculator. We emphasize the contrast with the U-shaped profit relationship between m and profits that emerges when the speculator is exogenously endowed with a fraction of liquidity order flow: when we endogenize the share m that the speculator processes, the added costs of processing additional order flow more than offset the possible direct trading benefits implied by the exogenous m, U-shaped profit relationship. The driving economic force is that a reduction in c raises m, which, in turn, feeds back to raise λ, which raises the incentive to process even more liquidity order flow; with the result that in the fixed point of the equilibrium, a reduction in c actually raises total expenditures cm^2 on processing liquidity order flow. In this way, costly processing of liquidity order flow always serves to reinforce the negative impact on speculator profits that we have identified. In particular, the speculator would *always* like to commit to processing a *smaller* fraction of liquidity order flow.

2.3. What about endogenous liquidity trade?

Finally, we investigate how outcomes are affected when we endogenize liquidity trade. We follow Spiegel and Subrahmanyam (1992) and consider risk-averse liquidity traders who receive endowment shocks of the risky asset and seek to rebalance their portfolio by offsetting some of their risky position. We consider two types of liquidity traders: N type 1 liquidity traders submit their orders indirectly via a monopolist informed, dual-trading speculator, while N type 2 liquidity traders submit their orders directly to the market maker. Liquidity trader i receives an endowment w_i of the risky asset, which, for a type j liquidity trader, is independently and normally distributed with zero mean and variance σ_{ju}^2/N. The speculator sees the asset value $e_1 \sim N(0,\sigma_e^2)$ and handles order flow $\sum_{k=1}^N u_{1jk}$ from type 1 liquidity traders; while type 2 liquidity traders submit $\sum_{k=1}^N u_{2jk}$ directly.

We deviate slightly from the Spiegel and Subrahmanyam (1992) framework and assume that liquidity traders have mean–variance preferences over final wealth, W, seeking to maximize

$$U(W)=E[W]-\frac{\eta}{2}\mathrm{Var}[W],$$

where η is the coefficient of risk aversion. Spiegel and Subrahmanyam assume that liquidity traders have negative exponential preferences over final wealth, $-e^{-\eta W}$. With both sets of preferences, a liquidity trader's ex-post expected utility *given* any endowment shock is always well-defined, and, indeed, their ex-post optimization problems are identical, so that trading and pricing outcomes are the same. The reason that we focus on quadratic preferences is that with negative exponential preferences, a liquidity trader's ex-ante expected utility is[9]

$$-\sqrt{\frac{1}{1-2\eta\frac{\sigma_{ju}^2}{N}U(W)}},$$

which is only finite when liquidity traders are not too risk averse, i.e., when $2\eta(\sigma_{ju}^2/N)U(W)<1$. In contrast, with mean–variance preferences, ex-ante utility is always finite. From an ex-ante utility perspective, negative exponential preferences do not weigh negative and positive endowment shocks equally—for large positive endowment shocks, the expected utility consequences go to zero, while the impacts of negative endowment shocks are magnified by the weighting of the negative exponential preferences; and if $2\eta(\sigma_{ju}^2/N)U(W)\geq 1$, the high weighting of negative tail endowment shocks swamps the ex-ante utility calculation, delivering unboundedly negative utility. Because we want to sign universally the comparative static impacts of changes in primitives on ex-ante welfare, we focus on mean–variance preferences.

The timing is as follows: first the asset value e_1 and liquidity trader endowment shocks w_1 and w_2 are realized. Then liquidity traders determine their orders, after which the speculator submits his order. The market maker receives the net order flow and then sets the price. We focus on linear trading strategies and linear pricing. In particular, liquidity trader j adopts trading strategy $u_j=\rho_j w_j$. Then, using our previous analysis, the speculator's order is

$$x_1(e_1,u_1)=\frac{e_1}{2\lambda}-\frac{u_1}{2}.$$

Substituting these strategies into λ yields

$$\lambda=\left[\frac{\frac{\sigma_e^2}{4}}{\left(\frac{\rho_1}{2}\right)^2\sigma_{1u}^2+\rho_2^2\sigma_{2u}^2}\right]^{0.5}=\left[\frac{\sigma_e^2}{\rho_1^2\sigma_{1u}^2+4\rho_2^2\sigma_{2u}^2}\right]^{0.5}. \tag{6}$$

Liquidity trader 1's terminal wealth when he trades u_1 is

$$e_1(u_1-w_1)-\lambda\left(\frac{e_1}{2\lambda}+u_1+\sum_{i=2}^{N}u_{1i}-\frac{u_1}{2}-\sum_{i=2}^{N}\frac{u_{1i}}{2}+\sum_{j=1}^{N}u_{2i}\right)u_1,$$

where we distinguish between liquidity trader 1 and the other $N-1$ liquidity traders of type 1 as well as those of type 2. The associated mean and variance of the value of his final holdings are

$$-\lambda\frac{u_1^2}{2}\quad\text{and}\quad\sigma_e^2\left[\frac{u_1}{2}-w_1\right]^2+\lambda^2u_1^2\rho_2^2\sigma_{2u}^2+\lambda^2u_1^2\rho_1^2\frac{N-1}{4N}\sigma_{1u}^2.$$

Hence, liquidity trader 1 maximizes

$$-\lambda\frac{u_1^2}{2}-\frac{\eta}{2}\left(\sigma_e^2\left[\frac{u_1}{2}-w_1\right]^2+\lambda^2u_1^2\rho_2^2\sigma_{2u}^2+\lambda^2u_1^2\rho_1^2\frac{N-1}{4N}\sigma_{1u}^2\right). \tag{7}$$

[9]See, for example, Diamond (1985) or Medrano and Vives (2004).

The associated first-order condition describing liquidity trader 1's optimal order is

$$0=-\lambda u_1-\eta\left(\sigma_e^2\left[\frac{u_1-2w_1}{4}\right]+\lambda^2 u_1\rho_2^2\sigma_{2u}^2+\lambda^2 u_1\rho_1^2\frac{N-1}{4N}\sigma_{1u}^2\right),$$

which has solution

$$u_1=\frac{\frac{\eta}{2}\sigma_e^2}{\lambda+\eta\left(\frac{\sigma_e^2}{4}+\lambda^2\rho_2^2\sigma_{2u}^2+\lambda^2\rho_1^2\frac{N-1}{4N}\sigma_{1u}^2\right)}w_1.$$

Analogously, liquidity trader 2 maximizes

$$-\lambda u_2^2-\frac{\eta}{2}\left(\sigma_e^2\left[\frac{u_2}{2}-w_2\right]^2+\lambda^2 u_2^2\rho_1^2\frac{\sigma_{1u}^2}{4}+\lambda^2 u_2^2\rho_2^2\frac{N-1}{N}\sigma_{2u}^2\right), \tag{8}$$

with solution

$$u_2=\frac{\frac{\eta}{2}\sigma_e^2}{2\lambda+\eta\left(\frac{\sigma_e^2}{4}+\lambda^2\rho_1^2\frac{\sigma_{1u}^2}{4}+\lambda^2\rho_2^2\frac{N-1}{N}\sigma_{2u}^2\right)}w_2.$$

This confirms that equilibrium trading strategies are indeed linear. To solve for the equilibrium when it exists (liquidity traders are sufficiently risk averse that the market does not break down, i.e., η is large enough), we substitute for λ from Eq. (6) into the linear trading strategies of the liquidity traders. We see that when equilibrium exists, it is characterized by the solution to the following two simultaneous equations in ρ_1 and ρ_2:

$$\rho_1=\frac{\frac{\eta}{2}\sigma_e^2}{\left[\frac{\sigma_e^2}{\rho_1^2\sigma_{1u}^2+4\rho_2^2\sigma_{2u}^2}\right]^{0.5}+\eta\left(\frac{\sigma_e^2}{4}+\left[\frac{\sigma_e^2}{\rho_1^2\sigma_{1u}^2+4\rho_2^2\sigma_{2u}^2}\right]\left(\rho_2^2\sigma_{2u}^2+\rho_1^2\frac{N-1}{4N}\sigma_{1u}^2\right)\right)}, \tag{9}$$

$$\rho_2=\frac{\frac{\eta}{2}\sigma_e^2}{2\left[\frac{\sigma_e^2}{\rho_1^2\sigma_{1u}^2+4\rho_2^2\sigma_{2u}^2}\right]^{0.5}+\eta\left(\frac{\sigma_e^2}{4}+\left[\frac{\sigma_e^2}{\rho_1^2\sigma_{1u}^2+4\rho_2^2\sigma_{2u}^2}\right]\left(\rho_1^2\frac{\sigma_{1u}^2}{4}+\rho_2^2\frac{N-1}{N}\sigma_{2u}^2\right).\right)}. \tag{10}$$

A type 1 liquidity trader's ex-ante expected utility is

$$\begin{aligned}U_1(N)&\equiv-\left(\lambda\frac{\rho_1^2}{2}+\frac{\eta}{2}\left(\sigma_e^2\left(\frac{\rho_1}{2}-1\right)^2+\frac{N-1}{N}\lambda^2\rho_1^2\rho_2^2\frac{\sigma_{1u}^2}{4}+\lambda^2\rho_1^2\rho_2^2\sigma_{2u}^2\right)\right)\frac{\sigma_{u1}^2}{N}\\&\equiv\hat{U}_1(N)\frac{\sigma_{u1}^2}{N},\end{aligned}$$

and the ex-ante expected utility of type 2 traders is

$$U_2(N) \equiv -\left(\lambda\rho_2^2 + \frac{\eta}{2}\left(\sigma_e^2\left(\frac{\rho_2}{2}-1\right)^2 + \lambda^2\rho_2^2\rho_1^2\frac{\sigma_{1u}^2}{4} + \frac{N-1}{N}\lambda^2\rho_1^2\rho_2^2\sigma_{2u}^2\right)\right)\frac{\sigma_{u2}^2}{N}$$
$$\equiv \hat{U}_2(N)\frac{\sigma_{u2}^2}{N}.$$

One's instinct may be to conjecture that liquidity traders who submit their orders via the speculator will have a higher trading intensity because they face lower order execution costs. However, this reasoning misses the fact that they are also subject to more uncertainty about the execution price of their orders. Proposition 3 below shows that when $N=1$, the trading intensity of the type 1 liquidity trader exceeds that of his type 2 counterpart if and only if they are sufficiently risk tolerant: there is a critical level of risk aversion that determines which liquidity agent offsets a greater fraction of his endowment shock. Furthermore, at this critical level, we can solve for equilibrium trading intensities and pricing.

Proposition 3. *For $N=1$, in any equilibrium, the liquidity trader who submits his order indirectly via the speculator offsets a greater fraction of his endowment, i.e., $\rho_1 > \rho_2$, if and only if the liquidity traders are sufficiently risk tolerant,*

$$\eta < \overline{\eta} = \frac{24\sigma_{2u}^2}{[\sigma_e^2(\sigma_{1u}^2 + 4\sigma_{2u}^2)]^{0.5}(4\sigma_{2u}^2 - \sigma_{1u}^2)}.$$

At $\eta = \overline{\eta}$, we have

$$\rho_1 = \rho_2 = \frac{\sigma_{1u}^2 + 4\sigma_{2u}^2}{6\sigma_{2u}^2} \equiv \overline{\rho} \quad \textit{and} \quad \lambda = \frac{6\sigma_e\sigma_{2u}^2}{(\sigma_{1u}^2 + 4\sigma_{2u}^2)^{1.5}} \equiv \overline{\lambda}.$$

Further, expected speculator profits are

$$\overline{\pi} = \overline{\rho}^2\overline{\lambda}\left(\frac{\sigma_{1u}^2}{4} + \sigma_{2u}^2\right) = \frac{\sigma_e\sqrt{\sigma_{1u}^2 + 4\sigma_{2u}^2}(\sigma_{1u}^2 + 2\sigma_{2u}^2)}{12\sigma_{2u}^2}.$$

Proof. See the Appendix.

In particular, if $\sigma_{1u}^2 = \sigma_{2u}^2 = \sigma_u^2$, then $\overline{\eta} = 8/\sqrt{5}\sigma_e\sigma_u$, $\overline{\rho} = \frac{5}{6}$, $\overline{\lambda} = 6\sigma_e/5\sqrt{5}\sigma_u$, and $\overline{\pi} = \sqrt{5}\sigma_e\sigma_u/4$. The economics underlying why liquidity trader 1 has a higher trading intensity than liquidity trader 2 if and only if they are sufficiently risk tolerant is straightforward: because the speculator offsets half of liquidity trader 1's order, the price impact is halved, which, ceteris paribus, induces liquidity trader 1 to submit a more aggressive order; to see this, compare the first terms in Eqs. (7) and (8). However, liquidity trader 1 faces greater trading risk than liquidity trader 2. In particular, both liquidity traders face the same asset uncertainty risk (to see this, compare the second terms in Eqs. (7) and (8)); but liquidity trader 1 faces greater uncertainty about the execution price of his order because liquidity trader 2's order contributes more to the determination of the market-clearing price than trader 1' order, precisely *because* the speculator offsets half of trader 1's order (see the third and fourth terms in Eqs. (7) and (8)). Of course, if the liquidity traders are not too risk averse, the price impact effect dominates in determining relative trading intensities.

We next generalize the result in Proposition 3 to allow for N liquidity traders of each type, establishing that there always exists a critical risk aversion level, $\overline{\eta}$, such that the trading intensities of the two types of liquidity traders are the same. That is, liquidity traders who trade with the speculator have a higher trading intensity if and only if $\eta<\overline{\eta}$. Further, we provide a general welfare irrelevance result: at $\overline{\eta}$ the ex-ante welfare of the liquidity traders is exactly the same, and it exceeds the ex-ante utility that they would receive were dual trading banned.

Proposition 4. *Suppose $\sigma_{u1}=\sigma_{u2}=\sigma_u$. Then, at*

$$\overline{\eta}=\left[\frac{4}{3\sqrt{5}\sigma_u\sigma_e}\right](5N+1),$$

liquidity traders who submit orders indirectly via the dual trading speculator have the same trading intensity as those who submit orders directly to the market maker,

$$\rho_1=\rho_2=\overline{\rho}=\frac{5N}{5N+1},$$

each liquidity trader has the same ex-ante expected utility, i.e., $U_1(N)=U_2(N)$, and speculator profits are $(\sqrt{5}/4)\overline{\rho}\sigma_e\sigma_u$. The ex-ante expected utility of liquidity traders is strictly higher than if dual trading is banned, while speculator profits are strictly lower.

Proof. See the Appendix.

For example, when $N=1$, at $\overline{\eta}=8/\sqrt{5}\sigma_e\sigma_u$ and $\sigma_{1u}^2=\sigma_{2u}^2=\sigma_u^2$, the ex-ante expected utilities of the two liquidity traders both equal $(-7/3\sqrt{5})\sigma_u\sigma_e$. That is, at $\overline{\eta}$, from an ex-ante perspective, it does not matter whether a liquidity trader submits his order indirectly via a dual-trading speculator or directly to the market maker.

To illustrate the content of Propositions 3 and 4, and the extent to which they generalize, we now numerically solve for equilibrium outcomes when $\eta\neq\overline{\eta}$, and contrast them with their counterparts when dual trading is banned. We show that the analytical welfare irrelevance result at $\overline{\eta}$ established in Proposition 4 does *not* extend to other risk aversion levels. In particular, when endowment shocks have the same variance, the liquidity trader type with the higher *endogenous* trading intensity also has the higher ex-ante expected utility (i.e., if $\eta<\overline{\eta}$, then it is the liquidity trader who trades with the speculator, and if $\eta>\overline{\eta}$, it is the liquidity trader who trades with the market maker). Moreover, while a continuity argument immediately implies that both liquidity trader types benefit from dual trading in the neighborhood of $\overline{\eta}$, *for risk aversion levels far from $\overline{\eta}$, one liquidity trader type benefits from dual trading at the other's expense.*

We first consider a symmetric setting with $\sigma_{1u}=\sigma_{2u}=\sigma_e=1$, and $N=1$ and vary the extent of liquidity trader risk aversion. The qualitative findings illustrated in Table 1 are robust. For an equilibrium to exist, speculators must be sufficiently risk averse; in this example, existence requires $\eta>\sqrt{2}$. The first panel highlights that for low values of risk aversion, $\eta<8/\sqrt{5}$, the liquidity trader who trades via the dual-trading speculator offsets a higher fraction of his endowment shock than the other liquidity trader, while for higher values of risk aversion this relationship is reversed.

Turning to welfare, we see that the liquidity trader type with the greater trading intensity also has the higher ex-ante expected utility, as measured by $\hat{U}_j(N)$ (recall that ex-ante expected utility is $U_j(N)=\hat{U}_j(N)\sigma_u^2/N$); with $\sigma_u^2=N=1$, these two measures correspond.

Table 1
Welfare and profit with dual trading: ($\sigma_e^2 = \sigma_{u1}^2 = \sigma_{u2}^2 = 1.0$, $N=1$).

η	λ	ρ_1	Dual trade				No dual trade			
			ρ_2	π	$\hat{U}_1$	$\hat{U}_2$	λ	ρ	π	$\hat{U}$
1.5	6.913	0.100	0.052	0.036	−0.712	−0.730	4.499	0.076	0.046	−0.721
2.0	1.290	0.476	0.306	0.194	−0.762	−0.847	0.905	0.391	0.237	−0.805
2.5	0.825	0.662	0.508	0.303	−0.836	−0.932	0.611	0.579	0.409	−0.888
3.0	0.644	0.764	0.676	0.388	−0.927	−0.993	0.502	0.705	0.498	−0.971
3.5	0.548	0.826	0.814	0.456	−1.027	−1.038	0.445	0.795	0.561	−1.055
$\frac{8}{\sqrt{5}}$	0.537	0.833	0.833	0.466	−1.044	−1.044	0.438	0.806	0.570	−1.068
4.0	0.488	0.866	0.928	0.512	−1.134	−1.072	0.387	0.914	0.609	−1.221
20.0	0.287	1.014	1.668	0.871	−4.931	−1.662	0.285	1.239	0.876	−3.805

We focus attention on the welfare measure $\hat{U}_j(N)$ that controls for the volatility of the endowment shock because numerically we obtain a very general solution for the difference in this welfare measure between liquidity trader types:

$$\hat{U}_1(N) - \hat{U}_2(N) = \frac{\eta}{4}[\rho_1 - \rho_2].$$

That is, controlling for the endowment shock variance, the welfare difference is proportional to the coefficient of risk aversion times the difference in trading intensities.

Table 1 shows that, reflecting this result, for risk aversion levels far from $\overline{\eta}$, liquidity trader welfare outcomes from dual trading can be uneven. For low values of risk aversion, liquidity traders who use the dual trader *gain* relative to the situation with no dual trading, while liquidity traders who submit directly to the market lose. In the neighborhood of $\overline{\eta}$, both types of liquidity traders gain, and, in particular, for risk aversion in excess of $\overline{\eta}$, the type 2 liquidity trader gains, while the type 1 liquidity trader is eventually hurt by his opportunity to trade with the dual trader whenever he is sufficiently risk averse. Lastly, the numerical calculations reinforce the point that dual trading uniformly lowers speculator profits relative to when dual trading is banned.

Table 2 considers a setting where liquidity trader risk aversion is fixed at $\eta = 2$, but varies the volatility of endowment and the number of liquidity traders. The results again underscore the fact that welfare gains can be uneven: for this level of risk aversion, type-2 liquidity traders are harmed by the presence of dual trading, while type-1 traders are helped. Recall that the $\hat{U}_j(N)$ welfare measure presented controls for the volatility of the endowment shock: to obtain ex-ante expected utility, multiply $\hat{U}_j(N)$ by σ_{uj}^2/N. In particular, while $\hat{U}_j(N)$ is falling in N, ex-ante expected utility $(\sigma_{uj}^2/N)\hat{U}_j(N)$ is increasing, reflecting that each liquidity trader is exposed to less endowment uncertainty as the liquidity shock variance is divided among more traders.

Finally, we recall that the forecast error variance, which we proved is unaffected by liquidity trading in Proposition 2, is always $\sigma_e^2/2$ in this monopoly speculator setting. We thus re-emphasize that the structure of liquidity trading—liquidity trader preferences, the structure of endowment shocks, the number of liquidity traders, etc.—has *no* impact on the informational efficiency of the market.

Table 2
Welfare and profit with dual trading, $2N$ liquidity traders: ($\sigma_e^2 = 1.0, \eta = 2$).

N	σ_{1u}^2	σ_{2u}^2	Dual trade								No dual trade							
			λ	ρ_1	ρ_2	π	$\hat{U}_1$	$\hat{U}_2$	U_1	U_2	λ	ρ_1	ρ_2	π	$\hat{U}_1$	$\hat{U}_2$	U_1	U_2
1	0.5	1.5	0.914	0.540	0.419	0.274	−0.730	−0.791	−0.365	−1.186	0.805	0.399	0.452	0.311	−0.801	−0.774	−0.400	−1.161
	1.0	1.0	1.290	0.476	0.306	0.194	−0.762	−0.847	−0.762	−0.847	0.905	0.391	0.391	0.237	−0.805	−0.805	−0.805	−0.805
	1.5	0.5	1.412	0.486	0.272	0.177	−0.757	−0.864	−1.136	−0.432	0.805	0.452	0.399	0.311	−0.774	−0.801	−1.161	−0.400
2	0.5	1.5	1.191	0.464	0.315	0.210	−0.766	−0.855	−0.191	−0.641	1.014	0.337	0.353	0.247	−0.832	−0.823	−0.208	−0.617
	1.0	1.0	1.506	0.415	0.259	0.166	−0.787	−0.878	−0.394	−0.439	1.056	0.335	0.335	0.237	−0.833	−0.833	−0.416	−0.416
	1.5	0.5	1.737	0.390	0.228	0.144	−0.796	−0.890	−0.597	−0.223	1.014	0.353	0.337	0.247	−0.823	−0.832	−0.617	−0.208
10	1.0	1.0	1.677	0.376	0.232	0.149	−0.804	−0.896	−0.804	−0.896	1.177	0.300	0.300	0.212	−0.850	−0.850	−0.850	−0.850

2.4. What about dynamic settings?

One has to work hard in dynamic settings to overturn the result that handling liquidity trade harms speculators. If either liquidity trade or private information about asset values is revealed at the end of each period, the same qualitative predictions emerge, as future pricing is unaffected by trade on liquidity order flow at earlier dates. Hence, to overturn the result, one needs long-lived information about asset values and liquidity trade.

Bernhardt and Taub (2008) explore the effect of introducing these features to a two-date version of Rochet and Vila's (1994) monopolist speculator model. They show that the Rochet and Vila irrelevance result breaks down. If the speculator knows both current and future liquidity trade, then he gains from front-running, trading in the same direction ahead of the liquidity trader before reversing his position, and this allows him to smooth profit extraction over time, thereby raising profits. However, if, instead, the speculator only learns about liquidity trade contemporaneously, he is hurt by the opportunity to trade on past liquidity trade, as it *exacerbates* his uneven intertemporal extraction of profits.

Hence, even with a monopolist speculator who observes all liquidity trade, it is difficult to generate the result that speculative profits are increased by observing liquidity trade. Introducing multiple speculators who have heterogeneous trading intensities on liquidity trade presumably makes it harder still.

3. Conclusion

One's instinct might be that speculators would gain from the opportunity to process uninformed liquidity trade, and then trading on that information. We consider a very general model of speculative trade, and show that this instinct is misplaced: in equilibrium, the market maker anticipates that with dual trading, he will receive less uninformed liquidity trade, and sets prices that are more sensitive to net order flow. We prove that for generic correlations in uninformed liquidity trade, speculators collectively are *hurt* by the opportunity to dual trade on uninformed liquidity trade.

One's instinct might then be that since speculators are hurt by the opportunity to dual trade via the equilibrium impact on pricing, then it must be that more information ends up being revealed through prices. We show that this instinct is also misplaced: fixing speculator information about fundamentals, the information content of prices is completely unaffected by the correlation structure of liquidity trade, and the information about liquidity trade that speculators possess: prices are just as informative with dual trading as without.

We then show that endogenizing speculator information about fundamentals via costly investment in signal quality, or endogenizing the share of the liquidity order flow that the speculator chooses to process serves to reinforce these core findings. We find that the opportunity to dual trade reduces the information speculators acquire about fundamentals, reducing their profits and reducing the informativeness of prices. More remarkable, we find that expected speculator profits *rise* monotonically with the costs of processing liquidity order flow as long as speculators do not process all liquidity trade.

Finally, we endogenize the trading motives of retail liquidity traders and compute liquidity trader welfare. We document that in any equilibrium, the trading intensities and welfare of liquidity agents who trade indirectly with via a dual trader are higher than those who trade directly with the market maker if and only if liquidity traders are sufficiently risk tolerant. Further, we prove that in the neighborhood of the critical level of risk aversion that determines which liquidity agent type has a greater trading intensity, all liquidity traders benefit from the presence of the dual-trading speculator, and further that the speculator would earn higher profits if dual trading was banned. From the policy perspective of a social planner, the bottom line of our analysis is that dual trading can offer liquidity traders welfare benefits; and, fixing speculator information about fundamentals, there is no offsetting cost in terms of less informative prices.

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Appendix A

Proof of Proposition 3. To determine the sign of $\rho_1 - \rho_2$, we exploit the structure of Eqs. (9) and (10), substituting $\rho_1 = \rho_2 = \rho$, and signing the differences in the two denominators,

$$\left[\frac{\sigma_e^2}{\rho_1^2\sigma_{1u}^2 + 4\rho_2^2\sigma_{2u}^2}\right]^{0.5} + \eta\left[\frac{\sigma_e^2}{\rho_1^2\sigma_{1u}^2 + 4\rho_2^2\sigma_{2u}^2}\right]\rho_2^2\sigma_{2u}^2 - 2\left[\frac{\sigma_e^2}{\rho_1^2\sigma_{1u}^2 + 4\rho_2^2\sigma_{2u}^2}\right]^{0.5} - \eta\left[\frac{\sigma_e^2}{\rho_1^2\sigma_{1u}^2 + 4\rho_2^2\sigma_{2u}^2}\right]\rho_1^2\frac{\sigma_{1u}^2}{4}$$

$$= \eta\left[\frac{\sigma_e^2}{\sigma_{1u}^2 + 4\sigma_{2u}^2}\right]\sigma_{2u}^2 - \left[\frac{\sigma_e^2}{\rho^2\sigma_{1u}^2 + 4\rho^2\sigma_{2u}^2}\right]^{0.5} - \eta\left[\frac{\sigma_e^2}{\sigma_{1u}^2 + 4\sigma_{2u}^2}\right]\frac{\sigma_{1u}^2}{4}.$$

Dividing by $[\sigma_e^2/\sigma_{1u}^2 + 4\sigma_{2u}^2]^{0.5}$, we see that $\rho_1 > \rho_2$ if and only if:

$$\eta\left[\frac{\sigma_e^2}{\sigma_{1u}^2 + 4\sigma_{2u}^2}\right]^{0.5}\left[\frac{4\sigma_{2u}^2 - \sigma_{1u}^2}{4}\right] - \frac{1}{\rho} < 0 \iff \rho < \frac{1}{\eta}\left[\frac{\sigma_{1u}^2 + 4\sigma_{2u}^2}{\sigma_e^2}\right]^{0.5}\frac{4}{4\sigma_{2u}^2 - \sigma_{1u}^2} \equiv \rho^*.$$

To solve for $\overline{\eta}$, substitute ρ^* into (9) and solve

$$\frac{1}{\eta}\left[\frac{\sigma_{1u}^2 + 4\sigma_{2u}^2}{\sigma_e^2}\right]^{0.5}\frac{4}{4\sigma_{2u}^2 - \sigma_{1u}^2} = \frac{\frac{\eta}{2}\sigma_e^2}{\frac{1}{\rho^*}\left[\frac{\sigma_e^2}{\sigma_{1u}^2 + 4\sigma_{2u}^2}\right]^{0.5} + \eta\left[\frac{\sigma_e^2}{4} + \left[\frac{\sigma_e^2}{\sigma_{1u}^2 + 4\sigma_{2u}^2}\right]\sigma_{2u}^2\right]}$$

$$= \frac{\frac{1}{2}}{\left[\frac{1}{\sigma_{1u}^2 + 4\sigma_{2u}^2}\right]\frac{4\sigma_{2u}^2 - \sigma_{1u}^2}{4} + \left[\frac{1}{4} + \left[\frac{1}{\sigma_{1u}^2 + 4\sigma_{2u}^2}\right]\sigma_{2u}^2\right]}$$

$$= \frac{2}{\left[\frac{4\sigma_{2u}^2 - \sigma_{1u}^2}{\sigma_{1u}^2 + 4\sigma_{2u}^2}\right] + 1 + \left[\frac{4\sigma_{2u}^2}{\sigma_{1u}^2 + 4\sigma_{2u}^2}\right]}$$

$$= \frac{\sigma_{1u}^2 + 4\sigma_{2u}^2}{6\sigma_{2u}^2}.$$

Solving yields

$$\overline{\eta} = \frac{24\sigma_{2u}^2}{[\sigma_e^2(\sigma_{1u}^2 + 4\sigma_{2u}^2)]^{0.5}(4\sigma_{2u}^2 - \sigma_{1u}^2)}.$$

Substituting for $\overline{\eta}$ into (9), we can solve for $\rho_1 = \rho_2 = \overline{\rho}$:

$$\overline{\rho} = \frac{6(\sigma_{1u}^2 + 4\sigma_{2u}^2)\overline{\rho}\sigma_{2u}^2}{16\sigma_{2u}^4 - \sigma_{1u}^4 + 6\sigma_{2u}^2\overline{\rho}(\sigma_{1u}^2 + 2\sigma_{2u}^2)}.$$

Solving yields

$$\overline{\rho} = \frac{\sigma_{1u}^2 + 4\sigma_{2u}^2}{6\sigma_{2u}^2},$$

and substituting for $\overline{\rho}$ into λ, we can solve for

$$\overline{\lambda} = \frac{6\sigma_e\sigma_{2u}^2}{(\sigma_{1u}^2 + 4\sigma_{2u}^2)^{1.5}}.$$

Using expected speculator profits equal expected liquidity trader losses, we can solve for

$$\overline{\pi} = \overline{\rho}^2\overline{\lambda}\left(\frac{\sigma_{1u}^2}{4} + \sigma_{2u}^2\right). \qquad \square$$

Proof of Proposition 4. When $\sigma_{u1} = \sigma_{u2} = \sigma_u$, and $\rho_1 = \rho_2 = \overline{\rho}$ then

$$\overline{\lambda} = \frac{\sigma_e}{\sqrt{5}\overline{\rho}\sigma_u},$$

and we can solve for $\overline{\rho}$ as a function of N using Eqs. (9) and (10):

$$\overline{\rho} = \frac{\frac{\overline{\eta}\sigma_e}{2}}{\frac{1}{\overline{\rho}\sqrt{5}\sigma_u} + \overline{\eta}\sigma_e\left[\frac{1}{4} + \frac{1}{5}\left(1 + \frac{N-1}{4N}\right)\right]} = \frac{\frac{\overline{\eta}\sigma_e\sigma_u}{2}}{\frac{1}{\overline{\rho}\sqrt{5}} + \overline{\eta}\sigma_e\sigma_u\left[\frac{10N-1}{20N}\right]}$$

$$\Rightarrow \quad \overline{\rho} = \frac{\frac{\overline{\eta}\sigma_u\sigma_e}{2} - \frac{1}{\sqrt{5}}}{\left[\overline{\eta}\sigma_e\sigma_u\frac{10N-1}{20N}\right]}$$

and

$$\overline{\rho} = \frac{\frac{\overline{\eta}\sigma_e}{2}}{\frac{2}{\overline{\rho}\sqrt{5}\sigma_u} + \overline{\eta}\sigma_e\left[\frac{1}{4} + \frac{1}{5}\left(\frac{1}{4} + \frac{N-1}{N}\right)\right]} = \frac{\frac{\overline{\eta}\sigma_e\sigma_u}{2}}{\frac{2}{\overline{\rho}\sqrt{5}} + \overline{\eta}\sigma_e\sigma_u\left[\frac{5N-2}{10N}\right]}$$

$$\Rightarrow \quad \overline{\rho} = \frac{\frac{\overline{\eta}\sigma_u\sigma_e}{2} - \frac{2}{\sqrt{5}}}{\left[\overline{\eta}\sigma_e\sigma_u\frac{10N-4}{20N}\right]}.$$

Solving for the $\overline{\eta}$ that equates these two trading intensity expressions for $\overline{\rho}$ yields

$$\overline{\eta}\sigma_e\sigma_u = \frac{20N+4}{3\sqrt{5}} \quad \text{and} \quad \overline{\rho} = \frac{5N}{5N+1}.$$

Substituting for $\overline{\eta}$ and $\overline{\rho}$ into $[U_1(N)-U_2(N)]$ at $\overline{\eta}$ yields

$$U_1(N)-U_2(N) \propto -\frac{\overline{\lambda}\overline{\rho}^2}{2} - \frac{\overline{\eta}}{2}\overline{\lambda}^2\overline{\rho}^4\frac{1}{N}\sigma_u^2\left(\frac{1}{4}-1\right) = -\frac{\overline{\rho}\sigma_e}{2\sqrt{5}\sigma_u}\left[1-\overline{\rho}\overline{\eta}\frac{\sigma_e}{\sqrt{5}\sigma_u}\sigma_u^2\frac{3}{4N}\right].$$

Hence, the ex-ante expected utility difference is zero, as

$$\begin{aligned}
1-\overline{\eta}\sigma_e\sigma_u\overline{\rho}\frac{3}{4\sqrt{5}N} &= 1-\left(\frac{\overline{\eta}\sigma_e\sigma_u}{2}-\frac{1}{\sqrt{5}}\right)\frac{20N}{10N-1}\frac{3}{4\sqrt{5}N} \\
&= 1-\frac{3\sqrt{5}\overline{\eta}\sigma_e\sigma_u-6}{2(10N-1)} \\
&= 1-\frac{3\sqrt{5}\left(\frac{20N}{3\sqrt{5}}+\frac{4}{3\sqrt{5}}\right)-6}{2(10N-1)} \\
&= 1-\frac{20N-2}{2(10N-1)} = 0.
\end{aligned}$$

Thus, with dual trading, the ex-ante utilities of the two liquidity traders are the same, and we can simply solve for that of liquidity trader 2:

$$U^{\text{dual}}(N) = -\left(\lambda\rho^2 + \frac{\eta}{2}\left(\sigma_e^2\left(\frac{\rho}{2}-1\right)^2 + \lambda^2\rho^4\frac{5N-4}{4N}\sigma_u^2\right)\right)\frac{\sigma_u^2}{N}.$$

Substituting $\overline{\eta}$ and $\overline{\lambda} = \sigma_e/\sqrt{5}\sigma_u\rho$ in utility yields

$$U^{\text{dual}}(N) = -\left(\frac{\rho}{\sqrt{5}} + \frac{2(5N+1)}{3\sqrt{5}}\left(\left(\frac{\rho}{2}-1\right)^2 + \frac{\rho^2}{5}\frac{5N-4}{4N}\right)\right)\frac{\sigma_e\sigma_u}{N}. \tag{11}$$

When dual trading is banned, liquidity trader 1 solves

$$\max_{u_1} -\lambda u_1^2 - \frac{\eta}{2}\left[\sigma_e^2\left[\frac{u_1}{2}-\omega_1\right]^2 + \lambda^2u_1^2\rho_2^2\sigma_{2u}^2 + \lambda^2u_1^2\rho_1^2\frac{N-1}{N}\sigma_{1u}^2\right]$$

with solution

$$\begin{aligned}
\rho_1 &= \frac{\frac{\eta}{2}\sigma_e^2}{2\lambda + \eta\left(\frac{\sigma_e^2}{4} + \lambda^2\rho_2^2\sigma_{2u}^2 + \lambda^2\rho_1^2\frac{N-1}{N}\sigma_{1u}^2\right)} \\
&= \frac{\frac{\eta}{2}\sigma_e^2}{2\left[\frac{\frac{\sigma_e^2}{4}}{\rho_1^2\sigma_{1u}^2+\rho_2^2\sigma_{2u}^2}\right]^{0.5} + \eta\left(\frac{\sigma_e^2}{4} + \frac{\frac{\sigma_e^2}{4}}{(\rho_1^2\sigma_{1u}^2+\rho_2^2\sigma_{2u}^2)}\rho_2^2\sigma_{2u}^2 + \frac{\frac{\sigma_e^2}{4}}{(\rho_1^2\sigma_{1u}^2+\rho_2^2\sigma_{2u}^2)}\rho_1^2\frac{N-1}{N}\sigma_{1u}^2\right)}.
\end{aligned}$$

If $\sigma_{1u}^2 = \sigma_{2u}^2 = \sigma_u^2$, then at $\overline{\eta}$:

$$\rho_1 = \rho_2 = \rho = \frac{\frac{\eta}{2}\sigma_e^2}{\frac{1}{\sqrt{2}}\frac{\sigma_e}{\rho\sigma_u} + \frac{\eta}{4}\left(\sigma_e^2 + \frac{\sigma_e^2}{2} + \frac{\sigma_e^2}{2}\frac{N-1}{N}\right)} = \frac{\frac{\eta}{2}\sigma_e^2}{\frac{1}{\sqrt{2}}\frac{\sigma_e}{\rho\sigma_u} + \frac{\eta}{4}\frac{4N-1}{2N}\sigma_e^2}.$$

At $\overline{\eta} = [4/3\sqrt{5}\sigma_u\sigma_e](5N+1)$, solve

$$\rho = \frac{\left[\frac{2(5N+1)}{3\sqrt{5}\sigma_u\sigma_e}\right]\sigma_e^2}{\frac{1}{\sqrt{2}}\frac{\sigma_e}{\rho\sigma_u} + \left[\frac{(5N+1)}{3\sqrt{5}\sigma_u\sigma_e}\right]\frac{4N-1}{2N}\sigma_e^2} = \frac{\frac{2(5N+1)}{3\sqrt{5}}}{\frac{1}{\sqrt{2}}\frac{1}{\rho} + \left[\frac{(5N+1)}{3\sqrt{5}}\right]\frac{4N-1}{2N}}$$

for

$$\rho = \frac{\frac{2(5N+1)}{3\sqrt{5}} - \frac{1}{\sqrt{2}}}{\left[\frac{(5N+1)}{3\sqrt{5}}\right]\frac{4N-1}{2N}} = \frac{4(5N+1) - 3\sqrt{2}\sqrt{5}}{5N+1}\frac{N}{4N-1}.$$

We can then calculate utility:

$$U^{\text{no dual}}(N) = -\left(\lambda\rho^2 + \frac{\eta}{2}\left(\sigma_e^2\left(\frac{\rho}{2} - 1\right)^2 + \lambda^2\rho^4\frac{2N-1}{N}\sigma_u^2\right)\right)\frac{\sigma_u^2}{N}.$$

Substituting for $\lambda\rho = \sigma_e/2\sqrt{2}\sigma_u$ and $\overline{\eta} = [4/3\sqrt{5}\sigma_u\sigma_e](5N+1)$ yields

$$U^{\text{no dual}}(N) = -\left(\frac{\rho}{2\sqrt{2}} + \frac{2(5N+1)}{3\sqrt{5}}\left(\left(\frac{\rho}{2} - 1\right)^2 + \frac{\rho^2}{8}\frac{2N-1}{N}\right)\right)\frac{\sigma_e\sigma_u}{N}. \tag{12}$$

Recognizing that while the trading intensities with and without dual trading differ, in both cases at $\overline{\eta}$, they depend only on N, and do not depend on σ_e or σ_u; and comparing ex-ante utilities with and without dual trading (Eqs. (11) and (12)), we see that, as a result, σ_e and σ_u enter multiplicatively. Therefore, the sign of the difference in ex-ante expected utilities only depends on N. Moreover, this difference is a continuously and boundedly differentiable function of N. One can rewrite this difference as a function of $1/N \in [0, 1]$. Therefore, as the difference is a boundedly differentiable function of a variable that is bounded between zero and one, to sign the difference analytically, it suffices to sign the difference on a sufficiently fine grid, i.e., we can sign the ex-ante utility benefit from dual trading graphically (see Fig. 1). The fact that the difference in ex-ante utility rises with N (falls with $1/N$) reflects that the critical level of risk aversion, $\overline{\eta}$, is increasing linearly in N.

When dual trading is banned, speculator profits are

$$2\lambda\rho^2\sigma_u^2 = \frac{\rho\sigma_e\sigma_u}{\sqrt{2}}.$$

Thus, speculator profits are higher when dual trading is banned, as

$$\frac{\rho\sigma_e\sigma_u}{\sqrt{2}} > \frac{\sqrt{5}}{4}\overline{\rho}\sigma_e\sigma_u \Leftrightarrow \frac{4(5N+1) - 3\sqrt{2}\sqrt{5}}{5N+1}\frac{N}{4N-1}\frac{1}{\sqrt{2}} > \frac{\sqrt{5}}{4}\frac{5N}{5N+1}$$

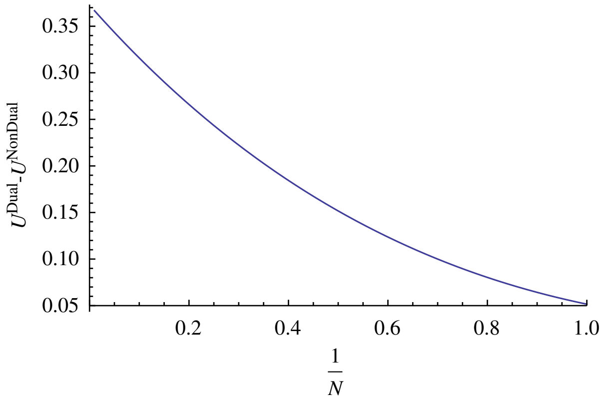

Fig. 1. Difference in ex-ante utility (with dual trading minus without) as a function of $1/N$.

$$\Leftrightarrow \frac{4(5N+1)-3\sqrt{2}\sqrt{5}}{4N-1}\frac{1}{\sqrt{2}} > \frac{\sqrt{5}}{4}5 \Leftrightarrow 16(5N+1)-12\sqrt{10} > 5\sqrt{10}(4N-1)$$
$$\Leftrightarrow (80-20\sqrt{10})N > 7\sqrt{10}-16,$$

which holds as $N \geq 1$. $\square$

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